

# MOEA/D with Adaptive IWO for Synthesizing Phase-Only Reconfigurable Linear Arrays

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**Abstract:** In order to well solve the phase-only reconfigurable arrays synthesis problems, we introduce an adaptive strategy in invasive weed optimization (IWO), and integrate the adaptive IWO (AIWO) into the framework of MOEA/D, a popular multi-objective algorithm. Then, a new version of MOEA/D with adaptive IWO, named MOEA/D-AIWO is proposed in this paper for solving the synthesis problems. In MOEA/D-AIWO, the proposed adaptive strategy is adopted for improving search ability and balancing diversity and convergence. We introduce an adaptive standard deviation, which changes not only with the increase of evolution generations, but also exponentially with the fitness function value of each individual. This strategy improves the convergence rate and helps the seeds escape from local optimum. Taking advantage of the powerful searching ability of invasive weeds and well framework of MOEA/D, the overall performance of the proposed MOEA/D-AIWO is illustrated in solving two sets of phase-only reconfigurable arrays synthesis problems. Comparing results with MOEA/D-IWO (MOEA/D with original IWO) and MOEA/D-DE are also provided in this paper.

**Keywords:** Array Synthesis, Adaptive, Invasive weed optimization, Multi-objective optimization, Phase-only.

## 1. INTRODUCTION

In many actual applications such as satellite communications and radar navigations, single antenna array is generally required to have the capability of producing a number of radiation patterns with different shapes, so as to save space and reduce cost. In practice, adjusting excitation phases is much easier than adjusting excitation amplitudes in the feeding network. Hence, phase-only reconfigurable array, which is designed to radiate multiple radiation patterns using a single power divider network and different phase shifters, attracts more and more attentions in recent years [1-10].

During the past decades, a number of design methods for synthesizing phase-only reconfigurable arrays have been proposed [1-10]. These methods can be divided into two categories. One is local search algorithms, like “alternating projections” methods [1-3], which are efficient and simple, but sensitive to initial values. The other is the evolutionary algorithms, such as genetic algorithm (GA) [4-6], particle swarm optimization (PSO) [6-8], differential evolution (DE) [9, 10] and so on, which have global searching ability. It has been shown that the evolutionary algorithms are more effective and flexible for synthesizing phase-only reconfigurable arrays [4-10]. More and more researchers prefer to adopt evolutionary algorithms for designing pattern reconfigurable arrays.

For pattern reconfigurable arrays, multiple patterns should meet their design indexes simultaneously, and different design pattern objectives often conflict with each other, then, the phase-only synthesis problem is actually a multi-objective optimization problem (MOP).

However, in most existing literatures for synthesizing pattern reconfigurable arrays, multiple objectives are usually summed into a single objective function with different weights, the multi-objective optimization problem is converted into a single-objective optimization problem. Although the optimization problem is solved easily by using this approach, some problems arise inevitably. The weights for different objectives depend on experience, thus decision-makers must have enough prior knowledge of the problem. Only when the weights are set properly, can the desired patterns be achieved. In order to get more optimal solutions, more experiments need to be done. These would be complex and time-consuming. Hence, in this paper, we intend to formulate the phase-only reconfigurable arrays design as a multi-objective optimization problem and solve it with our proposed multi-objective algorithm.

Multi-objective evolutionary algorithm based on decomposition (MOEA/D) proposed by Zhang and Li [11], is a well competitive algorithm for solving MOPs. MOEA/D makes use of the decomposition methods in mathematics and the optimization paradigm in evolutionary computation. Algorithms analysis demonstrates that [11], MOEA/D is of easy use and has a low complexity than NSGA-II, another classical multi-objective algorithm. Later, a new version of MOEA/D with differential evolution (DE) operator named MOEA/D-DE was proposed by Li and Zhang [12]. Experimental results show that MOEA/D-DE performs well on the MOPs with complicated Pareto set shapes. Thanks to MOEA/D's framework, theoretically, any evolutionary operator can be adopted in MOEA/D for actual problems.

Invasive weed optimization (IWO) is a numerical stochastic optimization algorithm inspired from weed coloniza-

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tion, which was proposed by Mehrabian and Lucas in 2006 [13]. The algorithm imitates seeds spatial diffusion, growth, reproduction, and competitive exclusion process of invasive weeds. With strong robustness and adaptability, IWO converges to the optimal solution effectively. It has been proved that IWO can successfully solve many single-objective optimization problems [14-18] and some MOPs [19, 20].

In order to solve the design problem of phase-only reconfigurable arrays effectively, a new version of MOEA/D with an adaptive IWO, named MOEA/D-AIWO is proposed. MOEA/D-AIWO under the structure of MOEA/D, decomposes the phase-only reconfigurable synthesis problem into a number of scalar subproblems and solves them simultaneously in a single run. At each generation, the population is composed of the best solutions found so far for each subproblem. Each subproblem adopts an adaptive IWO (AIWO) strategy for improving search ability. In AIWO, an adaptive standard deviation is proposed, which changes not only with the increase of evolution generations, but also exponentially with the fitness function value of each individual. This strategy improves the convergence rate and helps the seeds escape from local optimum. Taking advantage of the powerful searching ability of invasive weeds and well framework of MOEA/D, the overall performance of the proposed MOEA/D-AIWO is shown in solving the arrays synthesis problems. The arrays synthesis problems are also solved by MOEA/D-IWO and MOEA/D-DE, and comparing results are provided in this paper.

The remainder of this paper is organized as follows. Section 2 describes the adaptive strategy and IWO used in MOEA/D-AIWO. Section 3 presents basic principle of pattern reconfigurable arrays. Then in Section 4, detailed MOEA/D-AIWO for synthesizing phase-only reconfigurable linear arrays is introduced. Experimental results and discussions are given in Section 5. Finally, we conclude this paper in Section 6.

## 2. CLASSICAL IWO AND ITS ADAPTIVE MODIFICATION

The Invasive Weed Optimization (IWO) is a meta-heuristic algorithm that mimics the colonizing behavior of weeds. The flow of IWO may be summarized as follows:

**Step 1. Initialization:** A finite number of weeds are randomly initialized in the decision space.

**Step 2. Evaluate fitness and ranking:** Each initialized seed grows to a flowering plant. In other words, the fitness function returns a fitness value to be assigned to each plant and then these plants are ranked based on their assigned fitness values.

**Step 3. Reproduction:** Every plant produces seed based on its assigned fitness or ranking. The number of seeds each plant produces depends on the ranking of that plant and increases from minimum possible seed production to its maximum. This step adds an important property to the algorithm by allowing all of the plants to participate in the reproduction contest, it gives a chance to all agents to survive and reproduce based on their fitness.

**Step 4. Spatial Dispersion:** The produced seeds in this step are being dispersed over the search space by normally

distributed random numbers with mean equal to the location of producing plants and varying standard deviations. The standard deviation at the present time step can be expressed by:

$$\sigma_{\text{iter}} = \sigma_{\text{final}} + \left( \frac{\text{iter}_{\text{max}} - \text{iter}}{\text{iter}_{\text{max}}} \right)^{\text{pow}} (\sigma_{\text{initial}} - \sigma_{\text{final}}) \quad (1)$$

where  $\text{iter}_{\text{max}}$  is the maximum number of iterations.  $\text{iter}_{\text{initial}}$  and  $\text{iter}_{\text{final}}$  are defined initial and final standard deviations, respectively and  $\text{pow}$  is the nonlinear regulatory factor.

**Step 5. Repeat:** After this process carried out for all of the plants, the process is repeated at step 2. It should be noted that weeds with lower fitness are eliminated after ranking to reach the maximum number of plants in a colony.

In IWO, new solutions (seeds) produced are randomly distributed in D-dimensional space around their parents (weed) in normal distribution  $N(0, \sigma_{\text{iter}}^2)$ , the setting for  $\sigma_{\text{iter}}$  is expressed as Eq. (1). It can be seen from Eq. (1) that  $\sigma_{\text{iter}}$  decreases with the increase of iterations, while the value of  $\sigma_{\text{iter}}$  in one generation is the same. This is not conducive to the algorithm convergence. Generally speaking,  $\sigma_{\text{iter}}$  affects the distance between parent and her produced children weeds, different parent should have its own  $\sigma_{\text{iter}}$ , which is different with other parent weeds', though they are in the same generation. Thus, we modify IWO and present an adaptive standard deviation  $\text{std}_{\text{iter}}$ , in which the value of  $\sigma_{\text{iter}}$  in one generation changes not only with the iteration but also with the maximum, minimum and the individual's fitness value in the generation, as shown in the following equation:

$$\text{std}_{\text{iter}} = \begin{cases} e^{-\frac{\text{Fit} - \text{Fit}_{\text{mean}}}{\text{Fit}_{\text{max}} - \text{Fit}_{\text{mean}}}} \cdot \sigma_{\text{iter}} & \text{when } \text{Fit} \geq \text{Fit}_{\text{mean}}, \\ e^{\frac{\text{Fit}_{\text{mean}} - \text{Fit}}{\text{Fit}_{\text{mean}} - \text{Fit}_{\text{min}}}} \cdot \sigma_{\text{iter}} & \text{otherwise,} \end{cases} \quad (2)$$

where  $\text{Fit}$  is the fitness function value of the weed,  $\text{Fit}_{\text{max}}$ ,  $\text{Fit}_{\text{min}}$  and  $\text{Fit}_{\text{mean}}$  denote the maximum, minimum, and average fitness function values among all weeds in current generation, respectively.

It can be seen from Eq. (2) that, the adaptive standard deviation of the weed  $\text{std}_{\text{iter}}$  changes exponentially with its fitness value, and the higher the fitness value, the smaller standard deviation the weed will have, which enables the seeds distribute near around their better parents, and far away from their worse parents relatively. Moreover, the range of the adaptive standard deviation  $\text{std}_{\text{iter}}$  is  $[e^{-1}, e]$ , which makes the standard deviation of the weed in the

younger generations likely to be larger than that in the older generations. This will help the new produced seeds escape from local optimum, improve the convergence rate, and balance the global and local search capabilities effectively at the same time.

In the present generation, the number of seeds produced by the parent seed is calculated by

$$s_{\text{num}} = \text{floor} \left( \frac{\text{Fit} - \text{Fit}_{\min}}{\text{Fit}_{\max} - \text{Fit}_{\min}} (s_{\max} - s_{\min}) + s_{\min} \right) \quad (3)$$

where  $s_{\max}$  and  $s_{\min}$  are the largest and smallest numbers of seeds produced, respectively, and  $\text{floor}(\ast)$  represents the round-down function of “ $\ast$ ”. It is clear that better individuals produce more seeds.

Suppose  $x^i = (x_1^i, x_2^i, \dots, x_N^i)^T$ ,  $i = 1, 2, \dots, P$  is the present individual weed, and each new seed produced by  $x^i$  is  $y = (y_1, y_2, \dots, y_N)^T$ , in which each element  $y_k$  is generated as follows:

$$y_k = x_k^i + N(0, \text{std}_{\text{iter}}^2), k = 1, 2, \dots, N. \quad (4)$$

Then  $s_{\text{num}}$  new solutions are produced by Eqs. (1-4) and added into the population.

### 3. BASIC PRINCIPLE OF PATTERN RECONFIGURABLE ARRAY ANTENNAS

Design of phase-only reconfigurable antenna array is to find a common amplitude distribution and different phase distributions, such that the array can produce multiple different patterns.

Consider a linear equispaced array with  $N$  elements. If  $M$  different patterns need to be produced only by varying the excitation phases of the array under the common excitation amplitude distribution, the optimization variable  $x$  is a vector with  $MN + N$  elements, where  $x_n$  ( $n = 1, 2, \dots, N$ ) is the excitation amplitude for the  $n$ -th antenna element denoted by  $I_n$  and  $x_{mN+n}$  ( $n = 1, 2, \dots, N$ ) is the excitation phase for the  $n$ -th antenna element and the  $m$ -th pattern, denoted by  $\varphi_m$ . Then, the complex excitation of the  $n$ -th element in the  $m$ -th pattern is

$$i_{mn} = I_n \cdot e^{j\varphi_{mn}} = x_n e^{jx_{mN+n}}. \quad (5)$$

It can be seen from Eq. (5) that, in the process of optimization, the common excitation amplitude is used for  $M$  patterns all the time, and only the phases of the excitation are different. The  $m$ -th pattern produced by the antenna array for far field is given by

$$F_m(\theta) = \sum_{n=1}^N i_{mn} \cdot e^{2j\pi n d \cos\theta / \lambda}, \quad (6)$$

where  $m = 1, 2, \dots, M$ ,  $d$  is the spacing between array elements,  $\lambda$  is the wavelength in free space,  $\theta$  is the angle from ray direction to normal of array axis.

In this paper, patterns we need to reconfigure are presented below:

- (1) A cosecant-squared beam and a flat-top beam: the design problem is expressed as:

$$\min F(x) = (f_c(x), f_f(x))^T, \quad (7)$$

where

$$f_c(x) = \sum_{\theta=1}^{180} (\max\{|Q_\theta - Q_\theta(x)|, 0\})^2, \quad (8)$$

$$f_f(x) = \sum_{\theta=1}^{180} (\max\{|Q_\theta - Q_\theta(x)|, 0\})^2. \quad (9)$$

- (2) A cosecant-squared beam, a flat-top beam and a pencil beam: the design problem is formulated as:

$$\min F(x) = (f_c(x), f_f(x), f_p(x))^T \quad (10)$$

where

$$f_c(x) = \sum_{\theta=1}^{180} (\max\{|Q_\theta - Q_\theta(x)|, 0\})^2, \quad (11)$$

$$f_f(x) = \sum_{\theta=1}^{180} (\max\{|Q_\theta - Q_\theta(x)|, 0\})^2 \quad (12)$$

$$f_p(x) = \sum_{\theta=1}^{180} (\max\{|Q_\theta - Q_\theta(x)|, 0\})^2. \quad (13)$$

$f_c(x)$ ,  $f_f(x)$ ,  $f_p(x)$  represent the function of the cosecant-squared beam, flat-top beam and the pencil beam, respectively, and  $Q_\theta$ ,  $Q_\theta(x)$  are the desired and calculated values for each design specification we use. The lower the function value is, the closer the calculated pattern approaches the desired pattern. When the calculated values of all the indexes are lower than the corresponding desired values, the function value is set to zero.

### 4. MOEA/D WITH ADAPTIVE IWO (MOEA/D-AIWO)

MOEA/D decomposes the phase-only reconfigurable linear array synthesis problem into a number of scalar optimization subproblems and solves them in parallel. The objective in each of these subproblems is an aggregation of all the objectives in the array synthesis problem under consideration. Every subproblem has its own aggregation weight vector, which is different from any other subproblems, *i.e.*, all these aggregation weight vectors of the decomposed subproblems differ from each other. The number of the decomposed subproblems is also the population size. Suppose, is the population size, then, we need to optimize  $P$  subproblems in a single run.

There are several approaches for converting a pattern reconfigurable problem into a number of scalar optimization problems [21]. In our experiments, Tchebycheff approach is mainly employed. Let  $\lambda^1, \lambda^2, \dots, \lambda^P$  be a set of uniformly

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Input :  $P$ : the number of the subproblems;
           $T$ : the number of the weight vectors in the neighborhood of each weight vector,
           $0 < T < P$ ;
           $\lambda^1, \dots, \lambda^P$ : a set of  $P$  uniformly distributed weight vectors;
Output: Approximation to the PF:  $\{F(x^1), \dots, F(x^P)\}$ 

1 Initialization:
2 foreach  $i = 1$  to  $P$  do  $B(i) = \{a, b, \dots, t\}$ ; /*  $\lambda^a, \lambda^b, \dots, \lambda^t$  are the  $T$  closest
   weight vectors to  $\lambda^i$  */
3 initial population  $x^1, \dots, x^P$  by randomly sampling from  $[l, u]$ ,  $FV^i = F(x^i)$ ;
4 reference point  $z = (z_1, \dots, z_m)^T$ ; /*  $z_j = \min_{1 \leq i \leq P} f_j(x^i)$  */;
5 repeat
6 for  $i = 1$  to  $P$  do
7  $U \leftarrow \text{AIWO}(x^i, \text{std}_{iter}^i)$ ;
8  $V \leftarrow \text{AIWO}(x^k, \text{std}_{iter}^k)$ ; /*  $k$  is selected from  $B(i)$  */;
9 foreach  $y \in U \cup V$  do
10 if  $y \notin [l, u]$  then  $y \leftarrow \text{Repair}(y)$ ;
11 foreach  $j = 1$  to  $m$  do
12 if  $z_j > f_j(y)$  then  $z_j = f_j(y)$ ;
13 end
14 foreach  $j \in B(i)$  do
15 if  $g^{te}(y|\lambda^j, z) < g^{te}(x^j|\lambda^j, z)$  then
16  $x^j = y$ ;
17  $FV^j = F(y)$ ;
18 end
19 end
20 end
21 end
22 until stop criteria are met;

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Algorithm 1: MOEA/D-AIWO

distributed weight vectors and  $z^*$  be the reference point, i.e.,  $z^* = (z_1^*, \dots, z_m^*)^T$ ,  $z_i^* = \min\{f_i(x) | x \in [l, u]\}$  for each  $i = 1, 2, \dots, m$ . With the Techebycheff approach, the objective function of the  $i$ -th subproblem is in the form [21]:

$$g^w(x|\lambda^i, z^*) = \max_{1 \leq j \leq m} \{\lambda_j^i |f_j(x) - z_j^*|\}, \quad (14)$$

where  $\lambda^i = (\lambda_1^i, \dots, \lambda_m^i)^T$ . MOEA/D-AIWO minimizes all these  $P$  objective functions simultaneously in a single run. In each subproblem, AIWO is adopted for searching. Each subproblem is optimized by using information only from its neighboring subproblems. Neighborhood relations among subproblems are defined based on the distance between their aggregation coefficient vectors. Detailed description of MOEA/D-AIWO is shown in the following Algorithm 1.

In the population, optimization variables for each individual are the common excitation amplitudes for all patterns and different phases for forming different patterns. The output of the algorithm is a Pareto set, in which each Pareto optimal solution corresponds to a phase-only reconfigurable array. In the obtained results, different weight coefficients corresponding to various patterns form a weight vector, each Pareto optimal solution corresponds to a weight vector, all these weight vectors we set are different from each other, then, each optimal solution in Pareto set is different in principle. That is to say, we

can obtain a population of different phase-only reconfigurable array designs by MOEA/D-AIWO in a single run, in each design, synthesized patterns set has different weight vector. In actual applications, decision-makers select a desired solution from the approximated PF or obey some standards for choosing the best compromise solution.

## 5. EXPERIMENTS

To demonstrate the performance of the proposed MOEA/D-AIWO in the design of phase-only pattern reconfigurable linear arrays, we carry out two sets of experiments, and compare the experimental results with those obtained by MOEA/D-DE [12] and MOEA/D-IWO [20].

(1) Two patterns reconfigurable arrays design: A cosecant-squared beam and a flat-top beam.

Consider a  $0.5\lambda$ -equispaced linear array with 16 isotropic elements to generate a flat-top beam and a cosecant-squared beam. The dimension of vector  $x$  in objective function (Eq. (7)) is 48, including 16 common amplitudes and 32 unknown phases.

(2) Three patterns reconfigurable arrays design: A cosecant-squared beam, a flat-top beam and a pencil beam.

Consider a  $0.5\lambda$ -equispaced linear array with 16 isotropic elements to generate a cosecant-squared beam, a flat-top beam and a pencil beam. The dimension of vector  $x$  in objective function (Eq. (10)) is 64, including 16 common amplitudes and 48 unknown phases.

In the optimizing process, excitation amplitudes range from 0 to 1, and phases are restricted from -180 to 180 degrees.

**Table 1.** Parameters used in three comparing algorithms.

Parameters	Comparing Algorithms		
	MOEA/D-AIWO	MOEA/D-IWO	MOEA/D-DE
$P$	201/300	201/300	201/300
$T$	$0.1P$	$0.1P$	$0.1P$
$\eta_r$	$0.01P$	$0.01P$	$0.01P$
$\delta$	0.7	0.7	0.7
$\sigma_{ini}$	0.1	0.1	0.1
$\sigma_{final}$	0.002	0.002	0.002
$s_{max}$	3	3	3
$s_{min}$	1	1	1
$pow$	3	3	—
$CR$	—	—	1
$F$	—	—	0.5
$\eta$	—	—	20
$P_m$	—	—	$1/n$

### 5.1. Parameters Setting

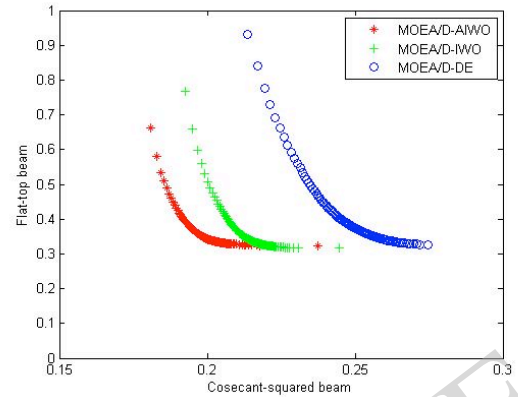
All the parameters used in MOEA/D-AIWO, MOEA/D-IWO and MOEA/D-DE are listed in Table 1. In MOEA/D, population size and weight vectors are controlled by an integer  $H$ .  $\lambda^1, \lambda^2, \lambda^k$  are the weight vectors in which each

dividual weight takes a value from  $\left\{ \frac{0}{H}, \frac{1}{H}, \dots, \frac{H}{H} \right\}$ , therefore

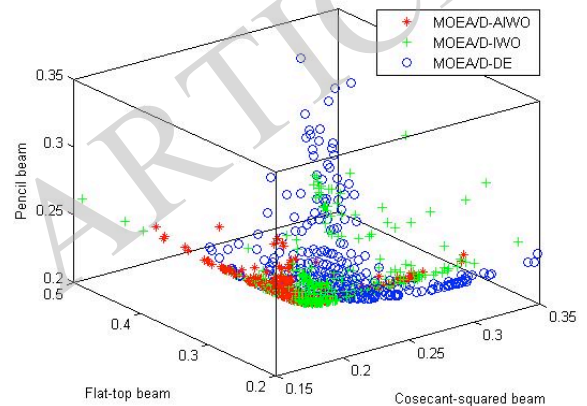
the population size  $P = C_{H+m-1}^{m-1}$ , where  $m$  is the number of objectives. In experiments,  $H$  is set to 200 and 23 for two-objective and three-objective instance respectively, therefore, the population size  $P$  is 201 for two-pattern and 300 for three-pattern reconfigurable arrays design. About the values setting of the other parameters in Table 1, we refer related references [12, 20]. Detailed descriptions can be found in [12, 20], we do not redescribe them here. To be fair in experimental comparisons, we set the maximum function evaluation 500000, and all these three algorithms (MOEA/D-AIWO, MOEA/D-IWO, and MOEA/D-DE) stop after reaching the maximum function evaluation.

### 5.2. Experimental Results and Analysis

In experiments, the synthesis problem is formulated as a multi-objective optimization problem, and solved by three comparing multi-objective algorithms: MOEA/D-AIWO, MOEA/D-IWO, and MOEA/D-DE. The output of these three algorithms for each instance is a Pareto set, in which each Pareto optimal solution corresponds to a phase-only reconfigurable array. Figs. (1) and (2) show the Pareto fronts of the two instances obtained by these three algorithms. It can be seen from the two figures that MOEA/D-AIWO performs the best.



**Fig. (1).** Plots of the final solutions obtained by MOEA/D-AIWO, MOEA/D-IWO and MOEA/D-DE for the first instance.



**Fig. (2).** Plots of the final solutions obtained by MOEA/D-AIWO, MOEA/D-IWO and MOEA/D-DE for the second instance.

The final approximations obtained by MOEA/D-AIWO have better spread and convergence than those obtained by MOEA/D-IWO and MOEA/D-DE in general, especially for the two-pattern. MOEA/D-IWO cannot obtain representative Pareto solutions within the given number of iterations. MOEA/D-DE performs the worst.

From the Pareto set, we choose the best compromise solution [20] obtained by each algorithm and present them in Figs. (3), (5) and Tables 2, 3. Fig. (3) and Table 2 are for instance 1, and Fig. (5) and Table 3 for instance 2. For the first instance, *i.e.*, the two-pattern reconfigurable array synthesis problem, it can be seen that, for the cosecant-squared beam the peak SLL obtained by MOEA/D-AIWO is 2.3554 dB lower than that obtained by MOEA/D-IWO and 0.5956 dB lower than that obtained by MOEA/D-DE. The BW value obtained by MOEA/D-AIWO is  $45^\circ$ , equal to that obtained by MOEA/D-IWO, while the value obtained by MOEA/D-DE is  $53^\circ$ , which is  $8^\circ$  wider than those obtained by MOEA/D-AIWO and MOEA/D-IWO. For the flat-top beam, it can be seen from Fig. (3) and Table 2 that, the pattern obtained by MOEA/D-AIWO has 0.9202 dB lower peak SLL value, which performs the best in those three algorithms. The SLL values obtained by MOEA/D-IWO and MOEA/D-DE are -20.5452 dB and -20.4597 dB, which are 0.375 dB and 0.4605 dB higher than that obtained by

MOEA/D-AIWO respectively. In terms of BW, MOEA/D-IWO performs the best. The BW value obtained by MOEA/D-IWO is  $141^\circ$ , which is narrower than  $145^\circ$ , that obtained by MOEA/D-AIWO and  $147^\circ$ , that obtained by MOEA/D-DE. Other indexes and the design objectives are listed in Table 2. Excitation amplitudes and the phases for the synthesized radiation patterns obtained by MOEA/D-AIWO are shown in Fig. (4).

In the second experiment, we use MOEA/D-AIWO to synthesize three patterns: A cosecant-squared beam, a flat-top beam and a pencil beam, and compare the results with those obtained by MOEA/D-IWO and MOEA/D-DE too. Fig. (5) and Table 3 show the desired values and the best compromise solutions obtained by three algorithms. It can be seen from Fig. (5) and Table 3 that, for the cosecant-squared beam the peak SLL obtained by MOEA/D-AIWO is 0.9903

Table 2. Design objectives and simulated results for the first instance.

	Desired	MOEA/D-AIWO	MOEA/D-IWO	MOEA/D-DE
<b>Cosecant-squared beam</b>				
Side lobe level (SLL, in dB)	-20	-22.3374	-19.9820	-21.7418
Half-power beam width (HPBW, in $\theta$ )	$30^\circ$	$34.9035^\circ$	$34.0448^\circ$	$35.1359^\circ$
Beam width at SLL (BW, in $\theta$ )	$40^\circ$	$45^\circ$	$45^\circ$	$53^\circ$
Ripple (in dB)	1	0.7002	1.2076	1.4299
<b>Flat-top beam</b>				
Side lobe level (SLL, in dB)	-20	-20.9202	-20.5452	-20.4597
Half-power beam width (HPBW, in $\theta$ )	$112^\circ$	$118.9857^\circ$	$118.9866^\circ$	$121.0613^\circ$
Beam width at SLL (BW, in $\theta$ )	$135^\circ$	$145^\circ$	$141^\circ$	$147^\circ$
Ripple (in dB)	1	0.9995	0.9995	0.9995

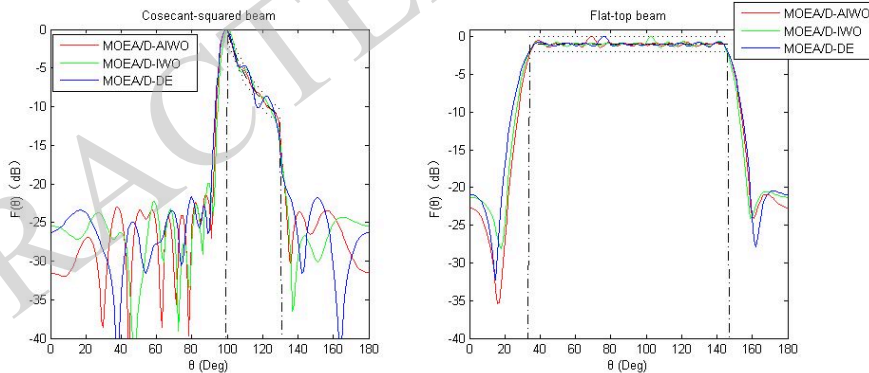


Fig. (3). Synthesized radiation patterns by MOEA/D-AIWO, MOEA/D-IWO and MOEA/D-DE for the first instance.

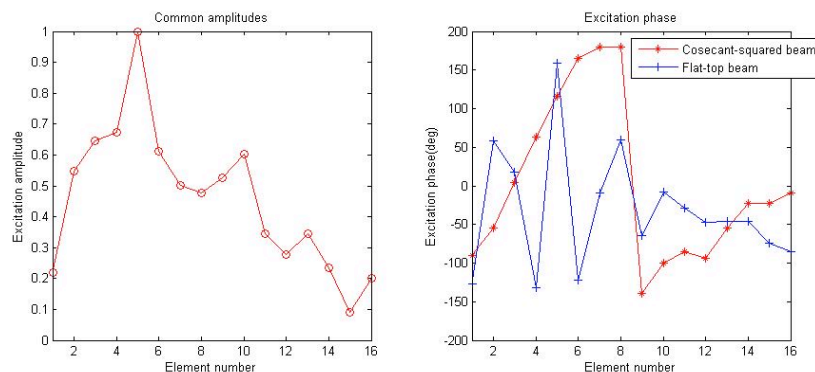


Fig. (4). Excitation coefficients for the synthesized radiation patterns obtained by MOEA/D-AIWO for the first instance.

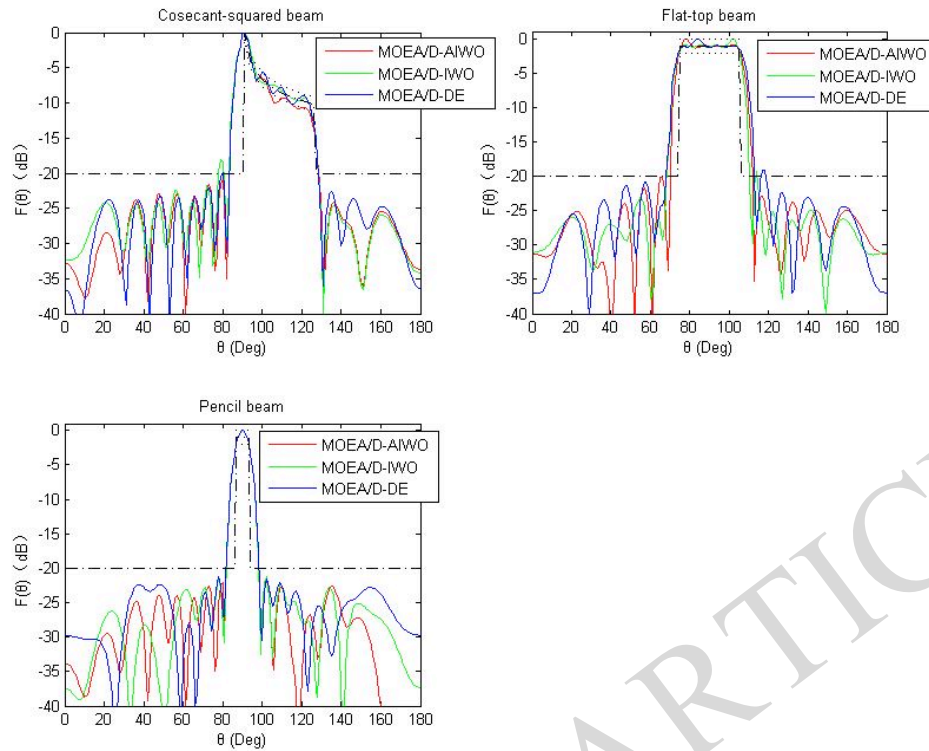


Fig. (5). Synthesized radiation patterns by MOEA/D-AIWO, MOEA/D-IWO and MOEA/D-DE for the second instance.

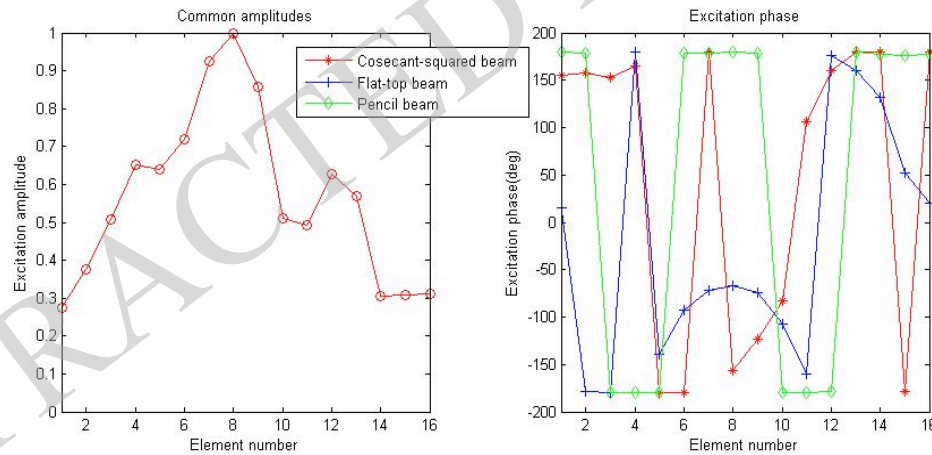


Fig. (6). Excitation coefficients for the synthesized radiation patterns obtained by MOEA/D-AIWO for the second instance.

dB lower than the desired objective value, while those obtained by MOEA/D-IWO and MOEA/D-DE are -18.095 and -19.9418 respectively, which have not met the design objective yet. All these three algorithms get the same  $49^\circ$  BW value. However, the ripple values obtained by three algorithms are all a bit large, they cannot meet the design objective well. For the flat-top beam, it can be seen from Fig. (5) and Table 3 that, MOEA/D-AIWO performs the best in every aspect. The SLL value obtained by MOEA/D-AIWO is -20.0552 dB, which is 0.7864 dB lower than that obtained by MOEA/D-IWO, and 0.9925 dB lower than that obtained by MOEA/D-DE. The BW value obtained by MOEA/D-AIWO is  $44^\circ$ , which is  $2^\circ$  narrower than that obtained by MOEA/D-IWO, and  $8^\circ$  narrower than that obtained by MOEA/D-DE. The ripple values obtained by three algorithms are all con-

trolled in 1 dB, which can meet the design objective. In the case of pencil beam, MOEA/D-AIWO also performs the best. In terms of SLL index, in three comparing algorithms, MOEA/D-AIWO gets the lowest value -21.5153 dB, which is 0.309 dB lower than that obtained by MOEA/D-IWO and 0.2957 dB lower than that obtained by MOEA/D-DE. In terms of BW, the value obtained by MOEA/D-AIWO is equal to that obtained by MOEA/D-IWO, and MOEA/D-DE performs the worst, the BW value obtained by MOEA/D-DE is  $20^\circ$ , which is  $2^\circ$  wider than those obtained by MOEA/D-AIWO and MOEA/D-IWO. Other indexes such as ripple value can meet the design objectives well. Excitation amplitudes and the phases for the synthesized radiation patterns obtained by MOEA/D-AIWO are shown in Fig. (6).

**Table 3. Design objectives and simulated results for the second instance.**

	Desired	MOEA/D-AIWO	MOEA/D-IWO	MOEA/D-DE
Cosecant-squared beam				
Side lobe level (SLL, in dB )	-20	-20.9903	-18.0950	-19.9418
Half-power beam width (HPBW, in $\theta$ )	35°	39.5926°	39.4687°	39.3322°
Beam width at SLL (BW, in $\theta$ )	45°	49°	49°	49°
Ripple (in dB)	1	2.4717	1.7150	1.7882
Flat-top beam				
Side lobe level (SLL, in dB )	-20	-20.0552	-19.2688	-19.0627
Half-power beam width (HPBW, in $\theta$ )	30°	34.6183°	34.6366°	36.5283°
Beam width at SLL (BW, in $\theta$ )	40°	44°	46°	52°
Ripple (in dB)	1	0.9995	0.9995	0.9995
Pencil beam				
Side lobe level (SLL, in dB )	-20	-21.5153	-21.2063	-21.2196
Half-power beam width (HPBW, in $\theta$ )	5°	9.4366°	9.3853°	9.4629°
Beam width at SLL (BW, in $\theta$ )	15°	18°	18°	20°
Ripple (in dB)	1	0.9995	0.9995	0.9995

All above experimental results show that MOEA/D-AIWO is competitive. Under the same conditions, MOEA/D-AIWO performs better or at least equal to MOEA/D-IWO and MOEA/D-DE in general, which demonstrates that the adaptive strategy proposed in this paper is effective in balancing diversity and convergence of the search, it is beneficial for stressing the performance of algorithm. MOEA/D-AIWO is effective and efficient in synthesizing phase-only patterns reconfigurable arrays.

## CONCLUSION

In order to synthesize the phase-only reconfigurable antenna arrays effectively, a new version of MOEA/D based on adaptive IWO, called MOEA/D-AIWO, is proposed in this paper. MOEA/D-AIWO under the structure of MOEA/D, decomposes the phase-only reconfigurable synthesis problem into a number of scalar subproblems and solves them simultaneously in a single run. In each subproblem, an adaptive IWO strategy is adopted for improving search ability. We introduced an adaptive standard deviation, which changes not only with the increase of evolution generations, but also with the fitness function value of each individual. This strategy improves the convergence rate and helps the seeds escape from local optimum. Taking advantage of the powerful searching ability of invasive weeds and well framework of MOEA/D, the overall performance of the proposed MOEA/D-AIWO is tested in solving the synthesis problems.

Two sets of experiments are carried out to illustrate the effectiveness of MOEA/D-AIWO, and the comparing results with MOEA/D-IWO, MOEA/D-DE show the superiority of MOEA/D-AIWO in solving this kind of antenna array synthesis problems.

## CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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